2024.M27

2024L003G1EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2024

Mathematics

Paper 1

Ordinary Level

Friday 7 June Afternoon 2:00 - 4:30

300 marks

Examination Number

Date of Birth

For example, 3rd February 2005 is entered as 03 02 05

Centre Stamp

The 2024 examination papers were adjusted to compensate for disruptions to learning due to COVID-19. This examination paper does not necessarily reflect the same structure and format as the examination papers of past or subsequent years.

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Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- any **five** questions from Section A – Concepts and Skills
- any **three** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

Answer **any five questions** from this section.

Question 1 **(30 marks)**

(a) A farmer buys a new machine for €30 000.
It depreciates (decreases in value) by 20% each year.

(i) Find the value of the machine after 1 year.

(ii) Hence, or otherwise, find the value of the machine after 2 years.

(b) A farmer bought some livestock for €4716, including VAT.
The cost of the livestock was €4500, excluding VAT.

Calculate the percentage VAT rate used.

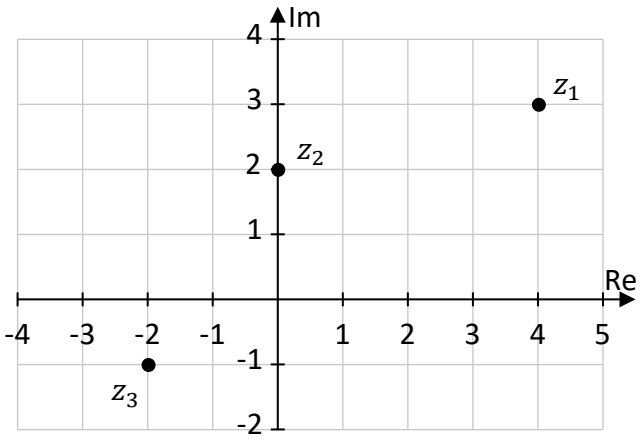
(c) The farmer also bought land.
One year later its value had increased by 12·5% and was now €52 875.

Find the value of the land when the farmer bought it.

Question 2 **(30 marks)**

In this question, $i^2 = -1$.

(a) The complex numbers z_1, z_2, and z_3 are shown on the Argand diagram below. One of these numbers is on the imaginary axis.



(i) Using the Argand diagram, write down the values of z_1, z_2 and z_3 in the form $a + bi$ where $a, b \in \mathbb{Z}$.

$z_1 =$ $z_2 =$ $z_3 =$

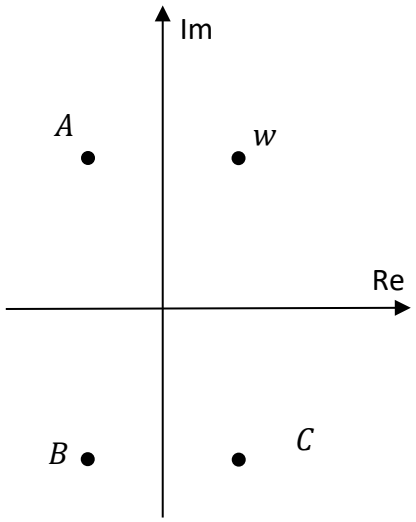
(ii) Write down the value of $|z_2|$.

$|z_2| =$ _____

(b) $v = \dfrac{15}{1+2i}$. Write v in the form $a + bi$, where $a, b \in \mathbb{Z}$.

(c) The complex number $w = 1 + 2i$ is shown on the Argand diagram below (drawn to scale).
The three complex numbers labelled A, B, and C are also shown on the Argand diagram.

One of these is $\overline{w}$, the complex conjugate of w.



Which of the complex numbers A, B or C is $\overline{w}$?
Give a reason for your answer.

$\overline{w} =$ (Tick (✓) **one** box only) A ☐ B ☐ C ☐

Reason:

Question 3 **(30 marks)**

(a) A toy shop sells two different sets of the same building blocks, Set **A** and Set **B,** as shown in the table below.

	Number of blocks	**Cost**
Set A	60	€12
Set B	150	€28

Which set of building blocks works out cheaper per block?
Use calculations to support your answer.

Answer: **Set A** **Set B**
(Tick (✓) **one** box only) ☐ ☐

Calculations:

(b) Solve the following equation in $x \in \mathbb{R}$:

$$3x^2 - 5x + 1 = 0$$

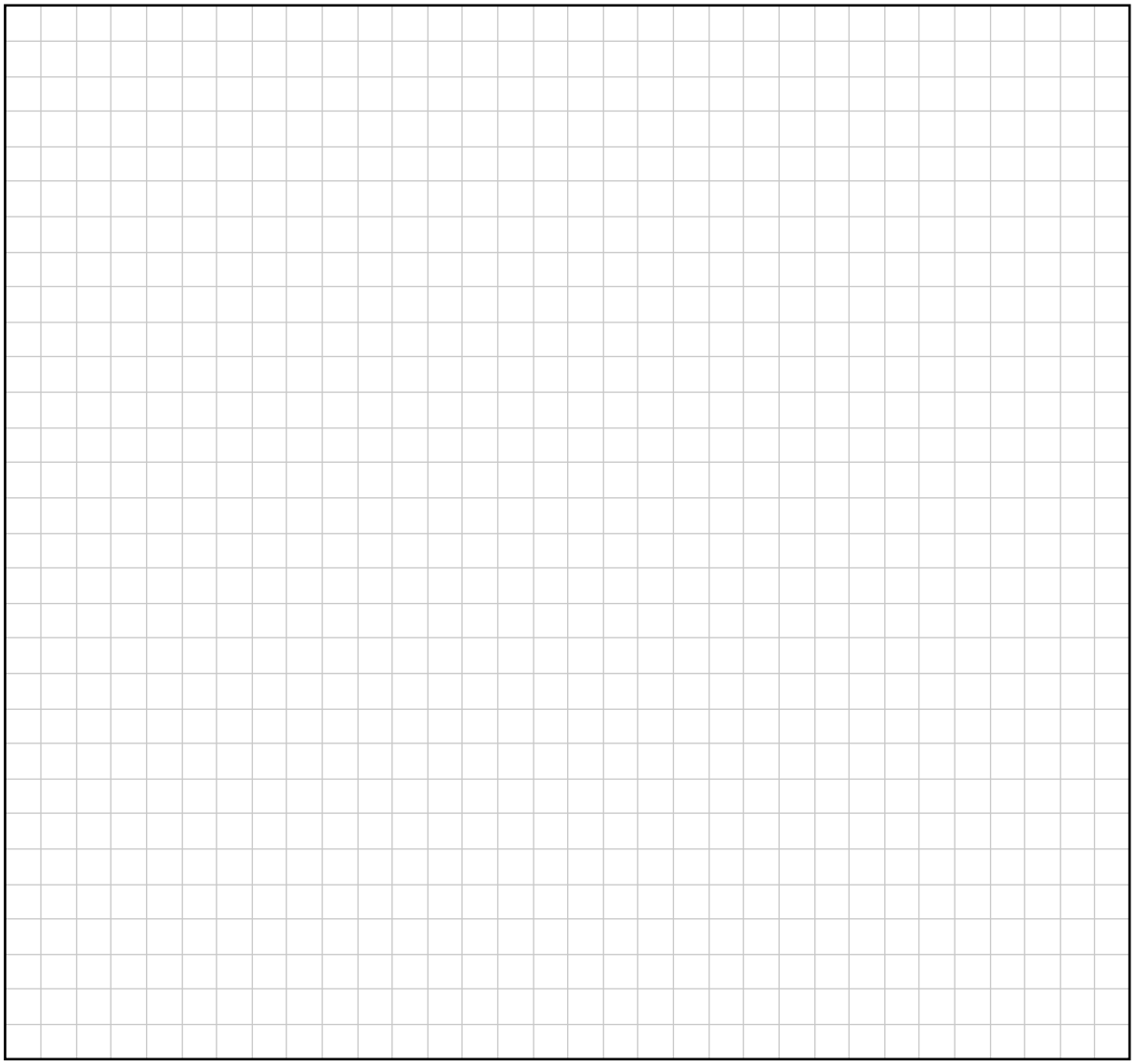
Give each answer correct to 2 decimal places.

(c) Use algebra to solve the simultaneous equations:

$$2x + y = 5$$

$$x^2 + y^2 = 25$$

Hint: first, use $2x + y = 5$ to write y in terms of x.



Question 4 **(30 marks)**

(a) Solve the following equation in $x \in \mathbb{R}$:

$$4(2x - 5) - 1 = 3x + 7$$

(b) Liam, Joan and Martha split a bill in the following ratio:

Liam		Joan		Martha
4	:	3	:	5

Liam and Joan paid €154 in total **between them**.
Martha paid the rest of the bill.

Work out how much Martha paid.

(c) **(i)** Write 128 in the form 2^k, where $k \in \mathbb{N}$.

(ii) Hence, or otherwise, solve the following equation, where $x \in \mathbb{R}$:

$$2^{4x+1} = 128$$

Question 5 **(30 marks)**

(a) Find the value of the following expression when $x = 2$.

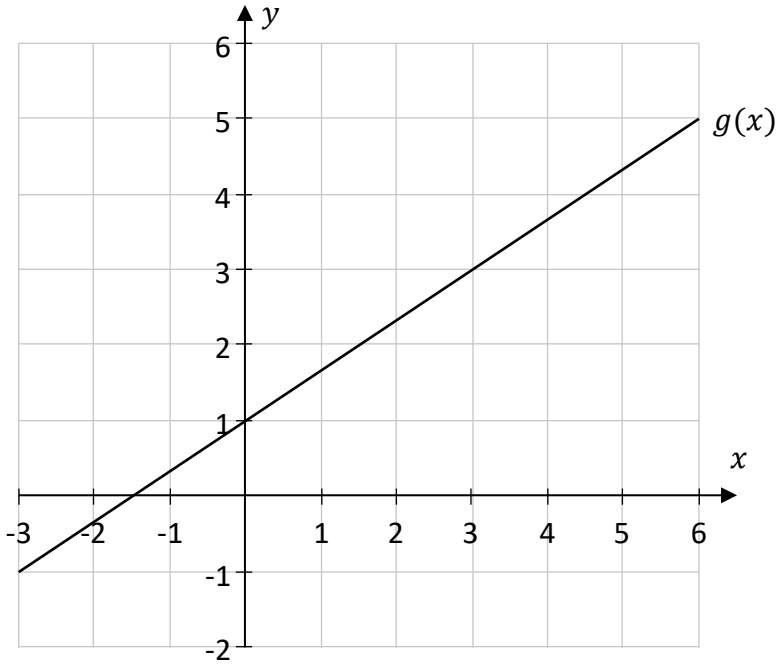
Give your answer in the form $\frac{a}{b}$ where $a, b \in \mathbb{N}$.

$$\frac{3x + 5}{10} - \frac{1}{x + 3}$$

(b) The function f is defined as $f(x) = 5x^2 - 20x + 2$, where $x \in \mathbb{R}$.

Use calculus to find the co-ordinates of the local minimum point of f.

(c) The graph of the linear function $g(x)$ is shown below.



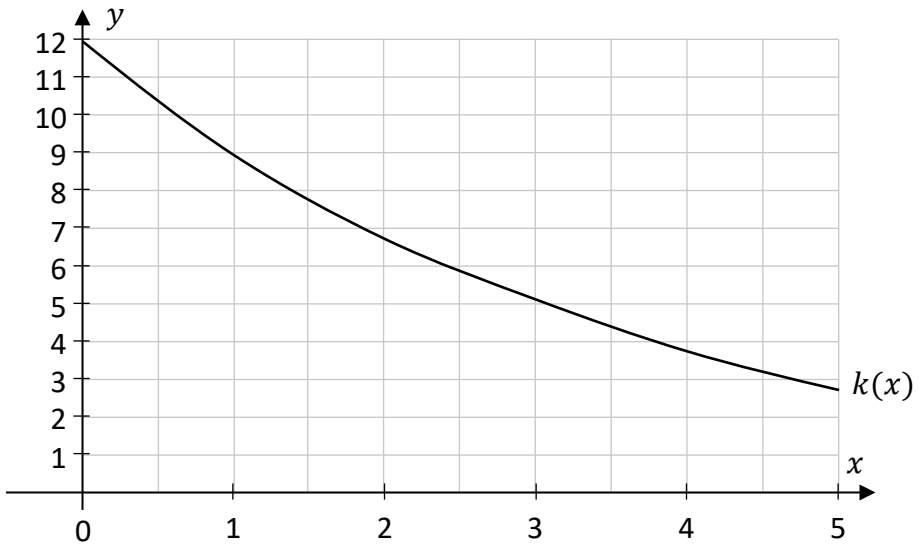
Use the graph to find the rate of change of $g(x)$, that is, find $g'(x)$.

Give your answer in the form $\frac{a}{b}$ where $a, b \in \mathbb{N}$.

Question 6 **(30 marks)**

The graph of a function $k(x)$ is shown on the co-ordinate diagram below, for $0 \leq x \leq 5,\ x \in \mathbb{R}$.

(a) Use the graph to answer **parts (a)(i)** and **(a)(ii)**.



(i) Estimate the value of $k(x)$ when $x = 2{\cdot}5$.
Show your work on the graph.

(ii) Estimate the range of values of x, for which $k(x) \leq 7$.
Show your work on the graph.

(b) Liam uses the **Trapezoidal Rule** to estimate the area under the graph of $k(x)$ between 0 and 5.

He takes the following measurements from the graph on the previous page:

x	0	1	2	3	4	5
$k(x)$	12	9	$6{\cdot}8$	$5{\cdot}1$	$3{\cdot}8$	$2{\cdot}8$

(i) Using the values of $k(x)$ in the table above, show that the Trapezoidal Rule will give an estimate of $32{\cdot}1$ units^2 for this area.

(ii) The actual area under $k(x)$, between 0 and 5, is $31{\cdot}8$ units^2. Liam estimated the area was $32{\cdot}1$ units^2 by using the Trapezoidal Rule.

Work out the **percentage error** in Liam's estimate.
Give your answer as a percent, correct to 2 decimal places.

Section B — Contexts and Applications — 150 marks

Answer **any three questions** from this section.

Question 7 **(50 marks)**

(a) During an experiment, the number of bacteria in a dish can be modelled by the function:

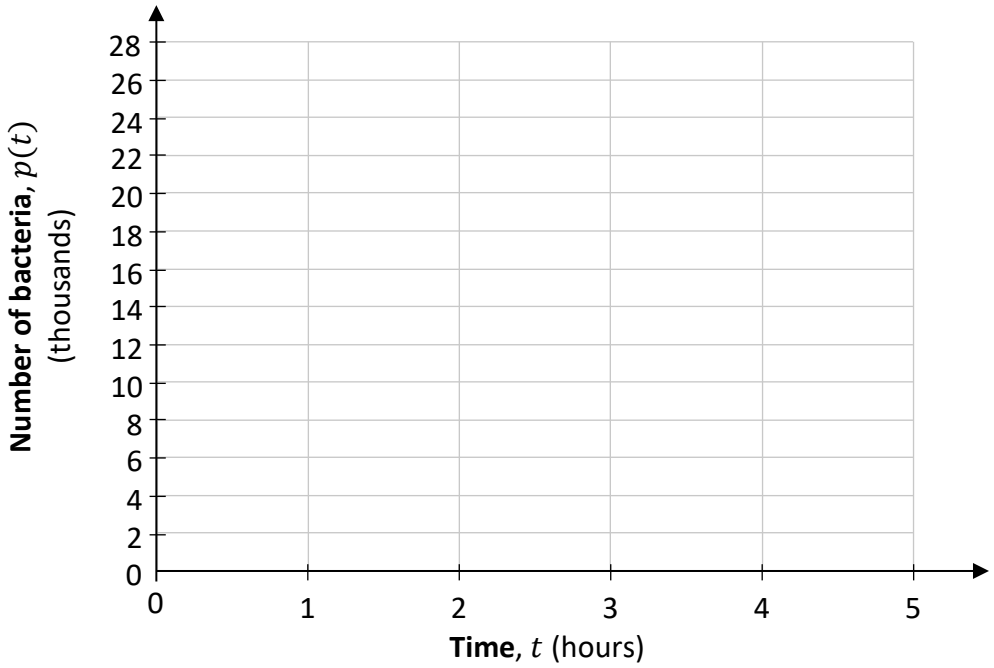
$$p(t) = t^3 - 6t^2 + 6t + 20$$

where $p(t)$ is the number of bacteria, in thousands, and t is the time in hours since the start of the experiment, with $0 \leq t \leq 5$, $t \in \mathbb{R}$.

(i) Complete the table below to show the values of $p(t)$ for the given values of t.

Time, t (hours)	0	1	2	3	4	5
Number of bacteria, $p(t)$ (thousands)			16		12	

(ii) Draw the graph of $p(t)$ on the axes below, for $0 \leq t \leq 5$, $t \in \mathbb{R}$.



(iii) Find $p'(t)$, the derivative of $p(t)$.
Remember that:

$$p(t) = t^3 - 6t^2 + 6t + 20$$

(iv) Find the value of $p'(2)$.

(v) $p'(4) = 6$.

Explain what this means in terms of the number of bacteria.

This question continues on the next page.

(b) The number of bacteria in a **different** experiment can be modelled by the function:

$$k(t) = 3000 \times 2{\cdot}72^{0{\cdot}5t}$$

where $k(t)$ is the number of bacteria and t is the time in hours since the start of the experiment, with $0 \leq t \leq 8$, $t \in \mathbb{R}$.

There are 3000 bacteria at the beginning of the experiment.

(i) Use $k(t)$ to find the number of bacteria after 1 hour **and** after 2 hours.
Give each answer correct to the nearest whole number.

After 1 hour

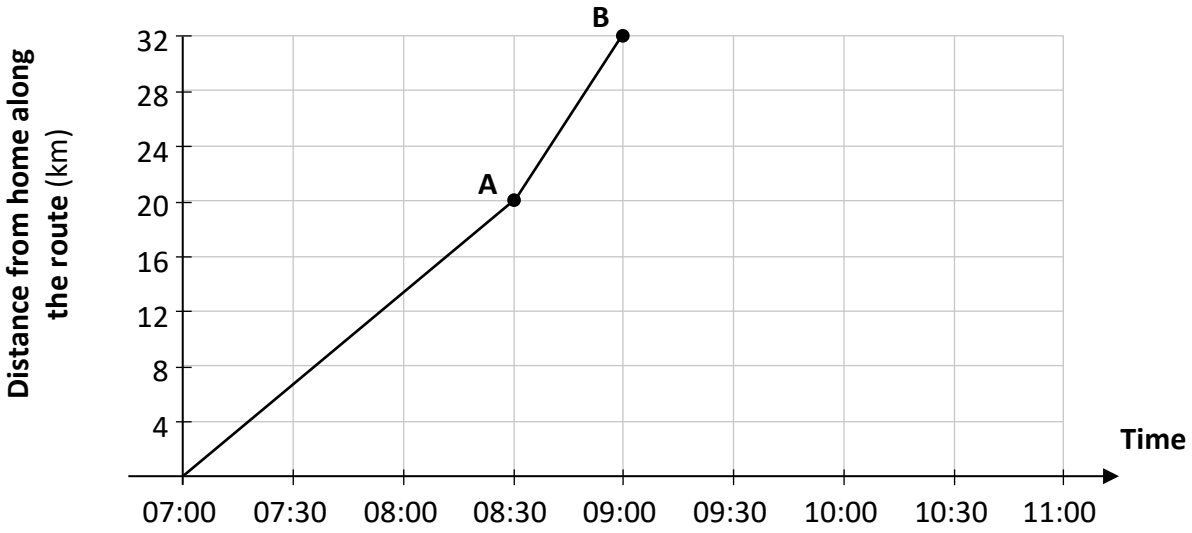
After 2 hours

(ii) After n hours, there are at least $35\,000$ bacteria according to $k(t)$, where $n \in \mathbb{N}$.
By continuing the pattern above, or otherwise, find the smallest possible value of n.

Question 8 **(50 marks)**

(a) Liam left home at $07:00$.
He cycled at a constant speed for 20 km, until he reached the point **A**. He then changed speed and cycled at a constant speed for a further 12 km, until he reached the point **B.**

The graph shows his distance from home along the route cycled until he reached **B**.



(i) Use the graph to estimate the distance Liam had cycled by $08:00$.
Show your work on the diagram above.

(ii) Find his speed in km/hour while he was travelling from **A** to **B**.

(iii) Use the following information to complete the graph on the diagram above to show the rest of Liam's journey:

- Liam stopped at **B** for 30 minutes
- He then cycled the 32 km back home at a constant speed.
  This took him 1 hour.

This question continues on the next page.

(b) Ellie takes up running. In the first week she runs 6 km.
Each week after this she runs $1{\cdot}5$ km more than the previous week.

(i) Complete the table below to show the distance Ellie runs each week for the first six weeks.

Week	1	2	3	4	5	6
Distance (km)	6	7·5				

(ii) If Ellie continued with this pattern, find the distance she would run in week 100.

(iii) Explain why Ellie cannot continue this pattern indefinitely.

(iv) S_n is the **total** distance, in km, run by Ellie after n weeks, where $n \in \mathbb{N}$.
Write S_n in terms of n.

(v) Using your answer to **part (b)(iv)**, or otherwise, find the **total** distance run by Ellie, in km, after 10 weeks if she continues the pattern (that is, find S_{10}).

Question 9 **(50 marks)**

(a) A ticket for a concert in Wales costs £59·50.

(i) Using the exchange rate €1 = £0·85, find the cost of the concert ticket, in euro.

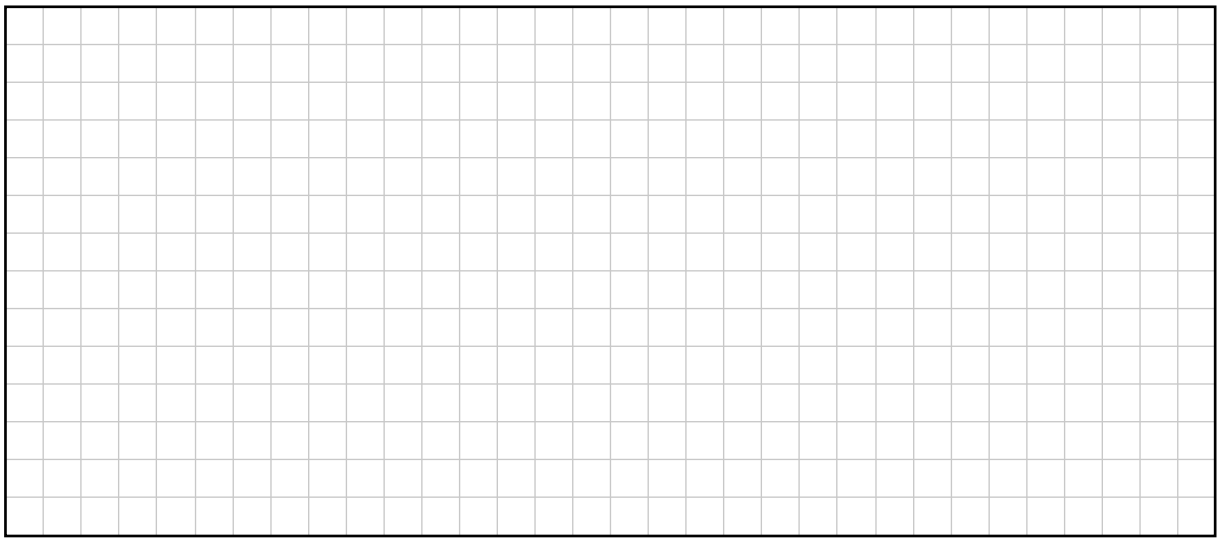
(ii) Liam is a member of the venue's *Loyalty Club*, so he gets a discount of 20% on all tickets. He buys 8 tickets for the concert.

Work out the total cost of the 8 tickets, in euro.

(b) Liam flies to the concert. He finds the following information:

- The aeroplane emits $28\,000$ grams of carbon per km, during the flight
- His total flight distance is 394 km
- There are 244 passengers on the flight.

Work out the amount of carbon emitted during the flight, per passenger.
Give your answer in kg, correct to 1 decimal place.



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The number of people who will attend a concert (the attendance) can change depending on the price of a ticket.

For a particular concert, this can be modelled by the function:

$$A(x) = 17\,600 - 160x$$

where $A(x)$ is the attendance at the concert and x is the price of a ticket, in euro, with $0 \leq x \leq 110$, $x \in \mathbb{R}$. Use the function $A(x)$ to answer **parts (c)(i)** and **(c)(ii)**.

(c) **(i)** Find the attendance at this concert if the price of a ticket is $€20$.

(ii) Find the price of a ticket (value of x) for which the attendance would be $12\,800$.

(d) For a different concert the total tickets sales, in euro, can be modelled by:

$$15\,000x - 150x^2$$

Again, x is the price of a ticket, in euro.

(i) For this concert the total tickets sales are €360 000.
Use this information to show that:

$$x^2 - 100x + 2400 = 0$$

(ii) Solve the equation $x^2 - 100x + 2400 = 0$ to find the two possible ticket prices for this concert.

Question 10 **(50 marks)**

(a) In 2023 Joseph has a gross annual income of €48 000.

He pays net annual income tax of €7650, annual PRSI of €1920, and pays €1407 in USC each year. He has no further deductions from his gross income.

(i) Work out his net annual income.

(ii) Hence, write his net annual income as a percentage of his gross annual income. Give your answer correct to the nearest percent.

(b) The two income tax rates in Ireland in 2023 are given in the table below.

Annual Income	Tax Rate
First €40 000	20%
Balance	40%

(i) Olivia's gross annual income is €34 000 and her annual tax credit is €3550. Find her net annual income.

(ii) Caroline's gross annual income is €50 000. Her annual tax credit is €3550. Find how much annual income **tax** she pays.

This question continues on the next page.

The two income tax rates in Ireland in 2023 are given again in the table below.

Annual Income	Tax Rate
First €40 000	20%
Balance	40%

(iii) The personal tax credit for an employee in Ireland in 2023 is €3550.
Find the greatest amount of income an employee can earn before paying any income tax.

Hint: The lowest income tax rate is 20%.

(iv) In 2024, the amount of income being taxed at the 20% rate was increased from €40 000 to €42 000.

Work out how much less income tax a person earning more than €42 000 would pay in 2024 compared to 2023 due to this increase.

(c) An employee who works from home may qualify for tax relief on their electricity and broadband bills.

The amount of tax relief such an employee can claim, A, is calculated using the formula:

$$A = \text{(Total annual electricity and broadband bills)} \times \frac{d}{365} \times 0{\cdot}3$$

where d is the number of days worked from home.

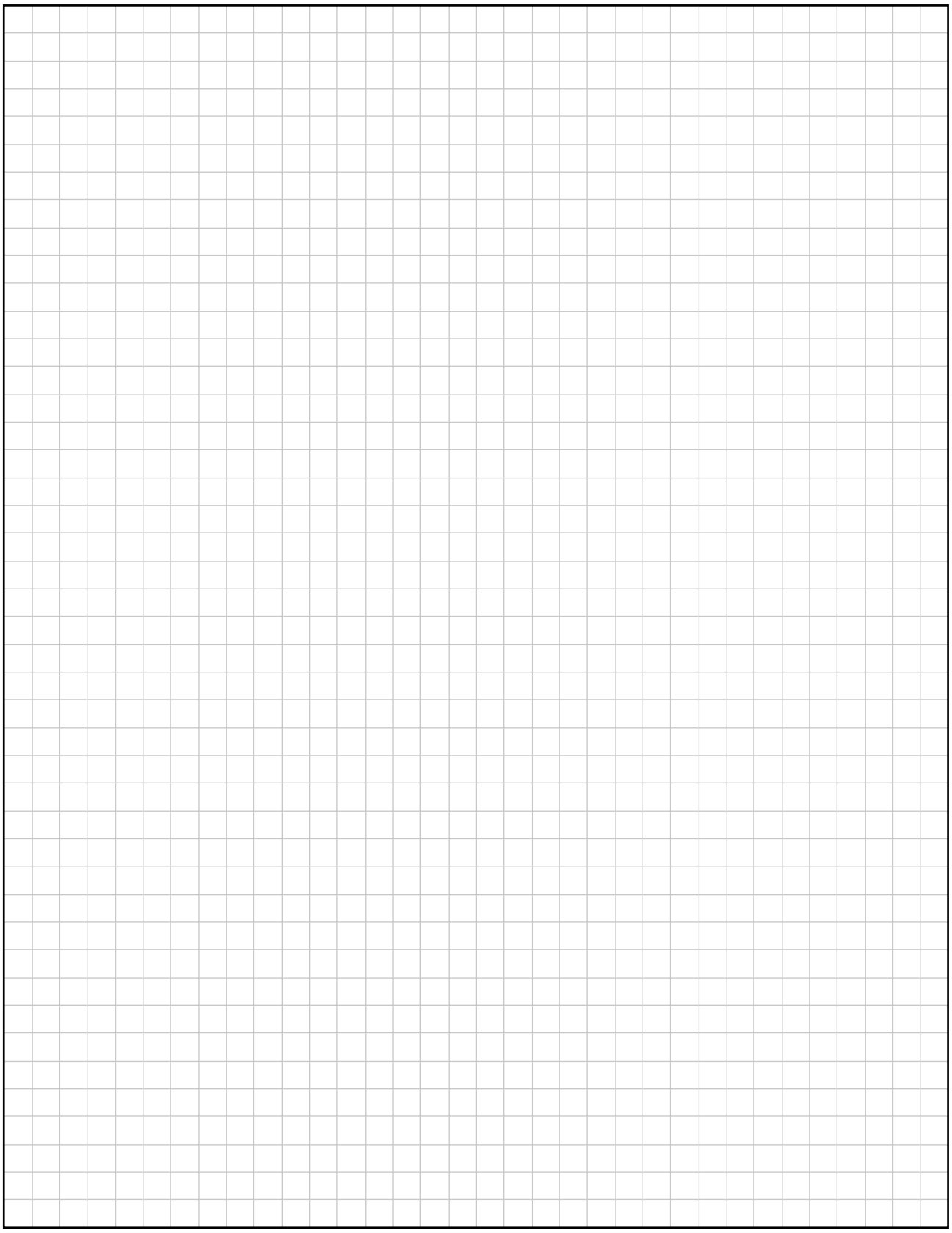
(i) Anne's annual electricity bill was $€1950$ and broadband bill was $€400$.
She worked from home for 120 days in the year.

Find how much tax relief she can claim.
Give your answer correct to the nearest cent.

(ii) Tara also worked from home for a number of days.
Tara's total electricity and broadband bills were $€2250$.
She claimed tax relief of $€135$ for working from home.

Calculate how many days she worked from home.

Page for extra work.
Label any extra work clearly with the question number and part.



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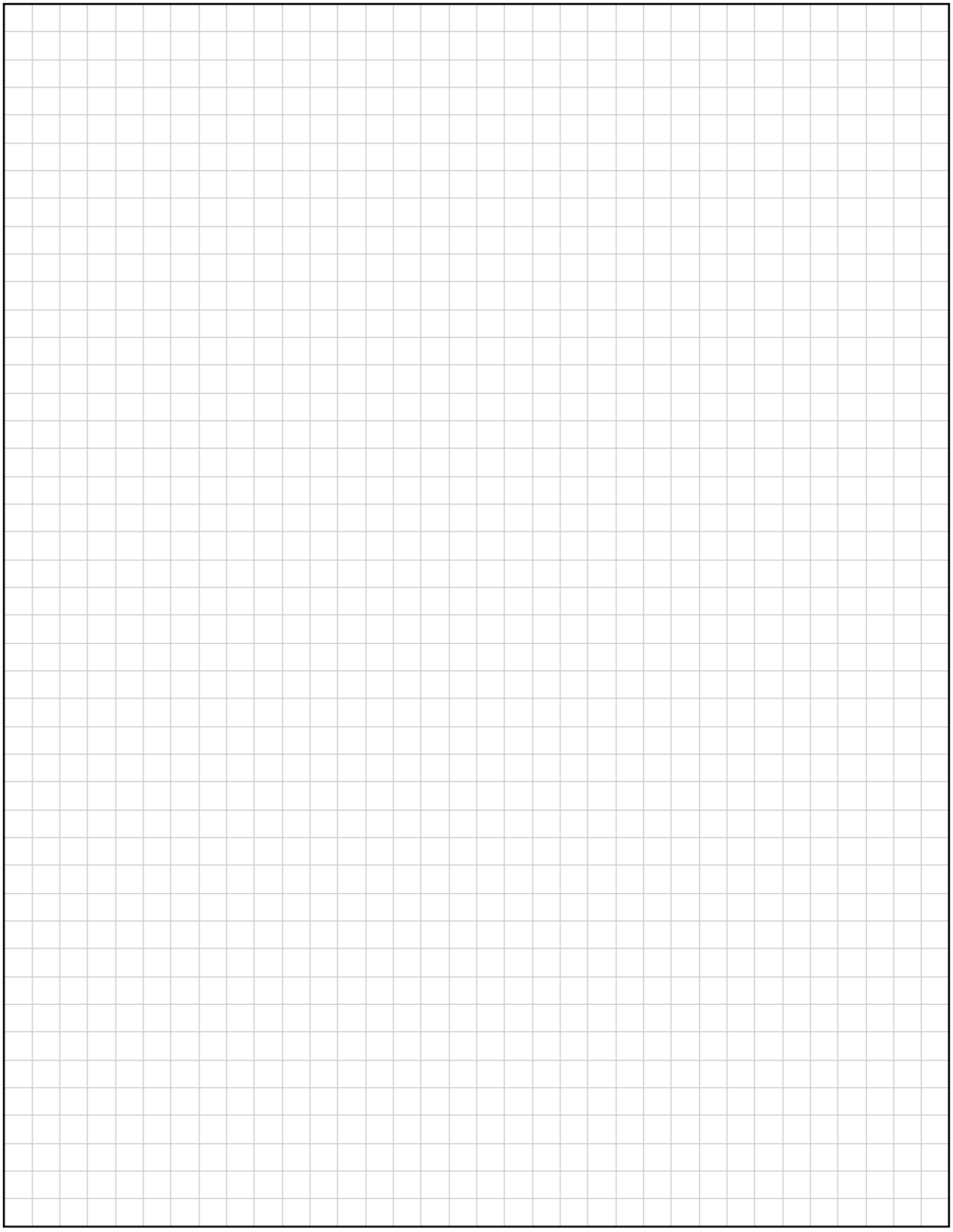

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Leaving Certificate – Ordinary Level

Mathematics Paper 1

Friday 7 June

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